Homework #6 – Due Nov. 5, 2015 by 3pm
(hand in to drop box outside 3L24)

1. From the definitions:

   $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ (vector operator "del"),

   $\nabla \varphi = \hat{i}\frac{\partial \varphi}{\partial x} + \hat{j}\frac{\partial \varphi}{\partial y} + \hat{k}\frac{\partial \varphi}{\partial z}$ (gradient of scalar function φ(x,y,z)), and

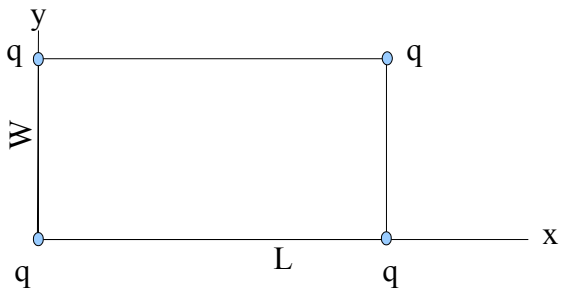
   $\nabla \cdot \vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

   (divergence of vector function $\vec{V}(x,y,z) = \hat{i}\,Vx(x,y,z) + \hat{j}\,Vy(x,y,z) + \hat{k}\,Vz(x,y,z)$), prove the following identities:

   (a) $\nabla(fg) = f\nabla g + g\nabla f$, where f and g are scalar functions.

   (b) $\nabla \cdot (f\vec{V}) = f\nabla \cdot \vec{V} + \vec{V}\nabla f$

2. Four identical point charges (q = +10 μC) are located on the corners of a rectangle as shown below with dimensions L = 60 cm, and W = 15 cm. Calculate the change in electric potential energy of the system as the charge at the lower left corner in the figure is brought to this position from infinitely far away. Assume that the other three charges remain fixed in position.

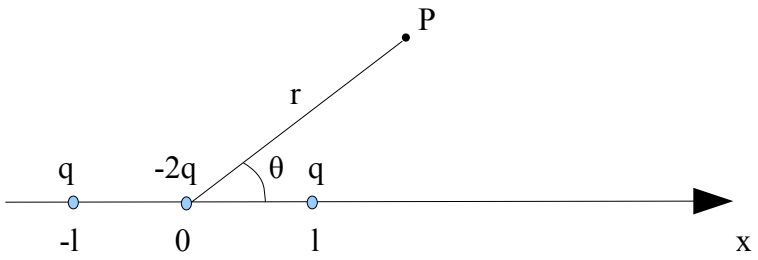


3. Show that the electric potential from an electric quadrupole, in the far field approximation (r>>l) is given approximately by (note: l is a lower case L):

$$V \approx k_e \frac{q\,l^2}{r^3}\left(3\cos^2\theta - 1\right)$$

   where θ is the angle between the x-axis and direction from the origin to point P. Recall, an electric quadrupole is a charge -2q at the origin, with two charges, each of +q on the x-axis at x = -l and x = +l respectively (see figure below).

   [Hint: Use the binomial theorem: $(1+s)^{-1/2} \approx 1 - \frac{1}{2}s + \frac{3}{8}s^2$, where s <<1]



4. An electron starts from rest 4 cm from the centre of a uniformly charged insulating sphere of radius 3 cm and total charge 2 nC. What is the speed of the electron when it reaches the surface of the sphere?

5. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of 120 V. (b) Calculate the speed of an electron that is accelerated through the same potential difference. (c) What potential difference is needed to stop an electron having an initial speed of 4.2×10^5 m/s?