PHYS-2201 Electricity and Magnetism
Assignment 15
Due: Tuesday, Feb. 23, 2015, by 3pm (at dropbox outside 3L24)

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1. (5 points) A copper solenoid is made by winding two layers of No. 14 copper wire on a cylindrical form 8 cm in diameter. There are four turns per centimetre in each layers, and the length of the solenoid is 32 cm. No. 14 copper wire has a diameter of 0.163 cm and resistance of 0.010 Ω/m at 75^oC (the coil will be hot). IF the solenoid is connected to a 50 V generator, what will the magnetic field strength at the centre of the solenoid be, and what is the power dissipation in watts?

2. (5 points) Consider a rotating insulating solid cylinder with radius, R and uniform charge density ρ throughout.
   (a) What is the magnetic field at a point on the axis of the cylinder if it is rotating about its axis with angular frequency ω?
   (b) How would your answer change if all of the charge were concentrated on the surface?

3. (5 points) Consider a material with a linear $\chi_m = 3.1$ in an external magnetic field $\vec{B_0} = 0.4y\hat{z}$ T. Find, as a function of y:
   (a) The magnetic field strength $\vec{H}$.
   (b) The material's magnetic moment, μ.
   (c) The relative magnetic moment μ_R relative to μ_0.
   (d) The magnetization of the material $\vec{M}$.
   (e) The current density of the applied field $\vec{J} = \nabla \times \vec{H}$.
   (f) The bound current in the material $\vec{J_b} = \nabla \times \vec{M}$.
   (g) The total effective current density.

4. (5 points) How should the current density inside an infinitely long thick cylindrical wire depend on r so that the magnetic field inside the wire has constant magnitude?

5. (5 points + 5 points bonus) *A valentines day bonus problem.* Sometimes the symmetry of a problem makes solving for the magnetic field too hard to do analytically; in that case it can be done numerically. Consider a loop carrying 100 A of current that follows the parametric function (in a clockwise sense):

$$\vec{r'} = 16\sin t\ \hat{x} + (13\cos t - 5\cos 2t - 2\cos 4t)\ \hat{y}\ \text{cm}, \tag{1}$$

   where $0 <= t < 2\pi$. In order to find the magnetic field due to this current loop, we can use following form of the Biot-Savart law:

$$d\vec{B} = \mu_0 I \frac{\vec{ds} \times (\vec{r} - \vec{r'})}{|\vec{r} - \vec{r'}|^3} = \mu_0 I \frac{\vec{ds} \times \vec{R}}{|R|^3}, \tag{2}$$

where $\vec{R} = (\vec{r} - \vec{r'})$ is the vector from the point we want to find the magnetic field, $\vec{r}$, and the location of the element of current $\vec{r'}$. The step around the current loop in the direction of the current is $\vec{ds}$. Use a spreadsheet to calculate the coordinates of the current loop at 40 values of t between 0 to 2π. From these 40 vector positions, calculate 39 values of $\vec{ds}$ vectors giving the direction of the current for the locations around the loop. Using $\vec{r} = \hat{x} + \hat{y} + \hat{z}$ cm as the position where we want to know the magnetic field, calculate $\vec{R}$ for each of the 39 current loop element locations, $\vec{ds} \times \vec{R}$, and $d\vec{B}$. Finally add up the 39 values of $d\vec{B}$ to get the overall magnetic field at this point. The columns of the spreadsheet could be: t, r'_x, r'_y, r'_z, ds_x, ds_y, ds_z, R_x, R_y, R_z, $|R|^3$, $(\vec{ds} \times \vec{R})_x$, $(\vec{ds} \times \vec{R})_y$, $(\vec{ds} \times \vec{R})_z$, dB_x, dB_y, dB_z, $|dB|$.