PHYS-2201 Electricity and Magnetism
Assignment 8
Due: Thursday, Nov. 19, 2015, by 3pm (at dropbox outside 3L24)

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1. (5 points) Two line charges, with line charge density λ, are separated by a distance L. A sketch looking down on the line charges, shown as points, is in Fig. 1. Show that in the far-field approximation ($r >> L$) that the potential at (r, θ) is given by:

$$\phi(r,\theta) \approx \frac{\lambda L}{2\pi\epsilon_0 r}\cos\theta. \tag{1}$$

Note that you cannot use the usual definition of $\phi = 0$ at ∞. Instead choose $\phi = 0$ at $r = 0$ (ie. for a single wire at $r = L/2$).

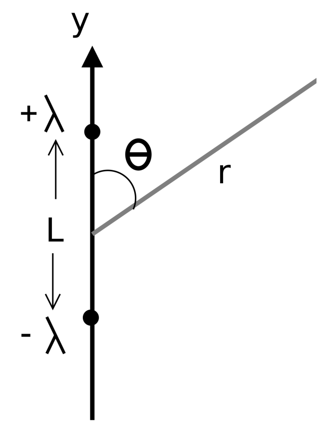

Figure 1: Problem 1.

2. (5 points) For the two line charges in Problem 1, in the far field approximation, find the electric field using $\vec{E} = -\nabla\phi$. In polar coordinates:

$$\nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta}. \tag{2}$$

3. (5 points) Find equations for the equipotentials and electric field lines for the two line charges of Problems 1 and 2. Sketch the equipotential and electric field lines.

4. (5 points) Near the surface of a disk of radius R and surface charge density σ has an electric field $E_\perp = \frac{\sigma}{2\epsilon_0}$ perpendicular to the surface. For a distance ηR along the y-axis, where $0 < \eta < 1$, show that the electric field parallel to the surface of the disk is given by:

$$E_\parallel = \frac{\sigma}{2\pi\epsilon_0}\int_0^{\pi/2} \ln\left(\frac{\sqrt{1-\eta^2\sin\theta^2}+\eta\cos\theta}{\sqrt{1-\eta^2\sin\theta^2}-\eta\cos\theta}\right)\cos\theta d\theta. \tag{3}$$

Hint: set up the integral as an integral over a wedge of charge as shown in Fig.2. Use symmetry to simplify the integral as you only need the vertical component of the field, and the integral over θ from 0 to π should be twice the integral from 0 to $\pi/2$.

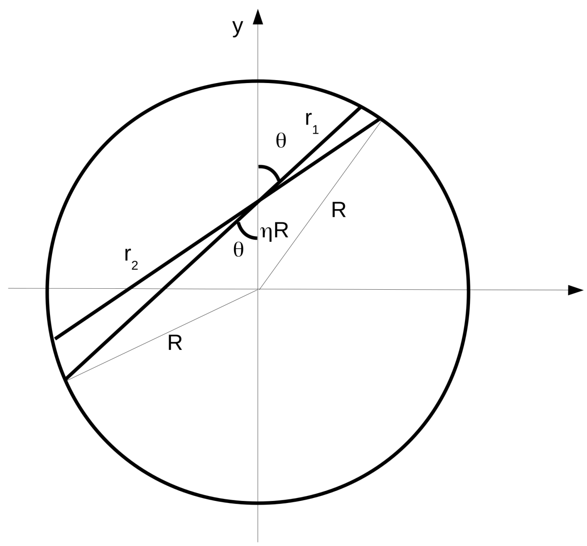

Figure 2: Problem 4.

5. (5 points) Using the result from problem 4, and with a liberal use of Taylor expansions to first order, show that as $\eta \to 0$, $E_{\|} \approx \frac{\sigma\eta}{4\epsilon_0}$. Also show that as $\epsilon = (1-\eta) \to 0$, $E_{\|} \approx -\frac{\sigma \ln \epsilon}{2\pi\epsilon_0}$.