Physics 2201 -- Midterm #1 Equation Sheet

$\omega = 2\pi f = \frac{2\pi}{T}$ $dq = \rho\, dV = \sigma\, dA = \lambda\, dl$

$x_f = x_i + v_i t + \frac{1}{2} a t^2$ $v_f = v_i + at$ $v_f^2 = v_i^2 + 2a(x_f - x_i)$

Electric Fields and Forces: $\sum \vec{F} = m\vec{a}$ $\vec{F} = q_0 \vec{E}$ $\Delta V = \frac{\Delta U}{q_0}$ $\Delta V = -E_0 d \, (for\ constant\ E)$

Work: $W = \Delta U = -q_0 \int_A^B \vec{E} \cdot \vec{ds}$ Electrostatic pressure: $p = \frac{\vec{F}}{A} = \frac{\sigma}{2}(\vec{E}_1 + \vec{E}_2)$

Point charges: $\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r}$ $\vec{E} = k_e \frac{q}{r^2} \hat{r}$ $\vec{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{r}_i$ $\vec{E} = k_e \sum_i \frac{q_i}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$

Continuous charges: $\vec{E} = \int d\vec{E} = k_e \int \frac{dq}{r^2} \hat{r} = k_e \int \frac{dq}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$

Along axis:

$E = \frac{\sigma}{2\epsilon_0} (plane)$ $E = k_e \frac{Qx}{(x^2 + a^2)^{3/2}} (ring)$ $E = 2\pi k_e \sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right] (disk)$

Dipole ($\vec{p} = q\vec{a}$): $\vec{E} = \frac{k_e}{r^3}[2p\cos\theta\, \hat{r} + p \sin\theta\, \hat{\theta}]$ $\vec{\tau} = \vec{p} \times \vec{E}$ $\sum \tau = I \frac{d^2\theta}{dt^2}$

Work: $W = -q \int_A^B \vec{E} \cdot \vec{ds}$ Simple harmonic motion: $\frac{d^2 x}{dt^2} = -\omega^2 x$

Gauss' Law: $\Phi_E = \oint_S \vec{E} \cdot \vec{dA} = \frac{q_{encl}}{\epsilon_0}$ Energy in electric field: $U = \frac{\varepsilon_0}{2} \iiint_{volume} E^2 dV$

Useful constants and integrals:

$e = -1.602 \text{x} 10^{-19} C$ $m_e = 9.11 \text{x} 10^{-31} kg$ $m_p = 1.67 \text{x} 10^{-27} kg$ $m_{deuteron} = 3.34 \text{x} 10^{-27} kg$

$N_A = 6.02 \text{x} 10^{23}$ $g = 9.8\, m/s^2$ $k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\, Nm^2/C^2$

$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, \ for\ n \neq -1$ $\int \cos x\, dx = \sin x + C$ $\int \sin x\, dx = -\cos x + C$

$\ln x + C, \ for\ n = -1$

$\int e^{ax} dx = \frac{e^{ax}}{a} + C$

Binomial expansion:

$(x + \Delta x)^n = x^n + n x^{n-1} \Delta x + \frac{n!}{2!(n-2)!} x^{n-2} \Delta x^2$ $(1+s)^{-1/2} \approx 1 - \frac{1}{2}s + \frac{3}{8}s^2 (s \ll 1)$

Review problems (taken from textbook, and past exams)

Refer to assignments 1-5, and CH. 23, 24 of S&J for additional problems

Shorter Problems

1. The electron gun in a television tube accelerates electrons (mass = 9.1×10^{-31} kg, charge = 1.6×10^{-19} C) from rest to 3.0×10^{7} m/s within a distance of 2.0 cm. What electric field is required?

2. An alpha particle (charge = $+2e$) is sent at high speed toward a gold nucleus (charge $+79e$). What is the electrical force acting on the alpha particle when it is at a distance of 2×10^{-14} m away from the gold nucleus? ($e = 1.6 \times 10^{-19}$ C)

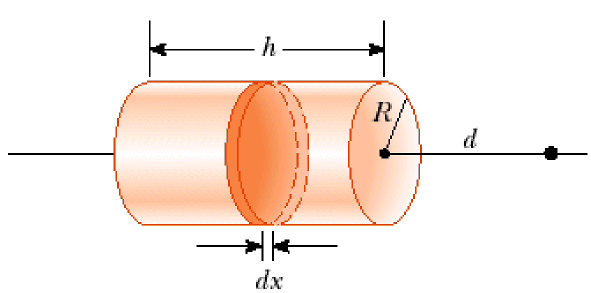
3. A proton moving at 3×10^{4} m/s is projected at an angle of 30° above a horizontal plane. If an electric field of 400 N/C is directed downwards, how long does it take the proton to return to the horizontal plane? (HINT: Ignore gravity) [$m_{\text{PROTON}} = 1.67 \times 10^{-27}$ kg, $q_{\text{PROTON}} = +1.6 \times 10^{-19}$ C.]

4. A sphere of radius 2a is made of non-conducting material that has a uniform volume charge density ρ. What is the electric field inside the cavity.

5. Three charges, each with charge +q, are located on the vertices of an equilateral triangle of side length a.
   (a) How much work does it take to move a charge +Q from infinitely far away to the center of the triangle?
   (b) How much work does it take to move the +Q from the center of the triangle to a point half way between two of the +q charges on the triangle?
   Assume the potential at ∞ is zero.

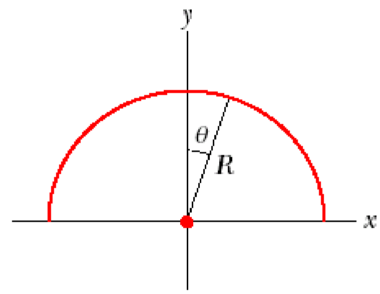
Longer Problems

6. Consider a uniformly charged thin-walled right circular cylindrical shell having total charge Q, radius R, and height h. Determine the electric field at a point a distance d from the right side of the cylinder as shown in Figure below. (*Suggestion:* Use the result for a ring charge and treat the cylinder as a collection of ring charges.) (b) **What If?** Consider now a solid cylinder with the same dimensions and carrying the same charge, uniformly distributed through its volume. Use the result for a disk of charge to find the field it creates at the same point.



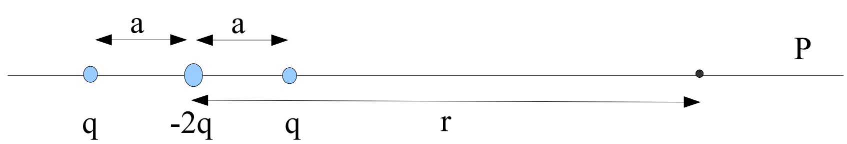
You may need the integral $\int \sqrt{x^2 \pm b^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm b^2} \pm \frac{b^2}{2}\ln\left(\left|x + \sqrt{x^2 \pm b^2}\right|\right) + C$

7. A line of positive charge is formed into a semicircle of radius R = 60.0 cm as shown in the figure below. The charge per unit length along the semicircle is described by the expression $\lambda = \lambda_0 \cos \theta$. The total charge on the semicircle is 12.0 μC. Calculate the total force on a charge of 3.00 μC placed at the center of curvature.



8. A uniformly charged semicircular arc of radius a and total charge Q lies in the upper half of the xy-plane, with its center at the origin. Calculate the work to move a charge q along the z-axis from $z = d$ to the origin.

9. A pair of electric dipoles is called a quadrupole. Find the electric field at a point P along the axis of the linear quadrupole shown in the figure below, at a distance r>>a from its centre.



10. Consider two co-axial disks, each of radius a, with a common z-axis. One lies on the $z = d/2$ plane, and has a uniform charge density σ. The other lies on the $z = -d/2$ plane, and has a uniform charge density -σ. Find the electric field along the z-axis.

11. An insulator in the shape of a half-disk of radius R, shown in the figure below 2 has a surface charge density $\sigma = \sigma_0 R r \sin \theta$, at a radius r from the origin and angle, θ from the x-axis. Find an expression for the Electric Field, **E**, at the origin.